Appendix A

For piezoelectric ceramics which exhibit symmetry of a hexagonal crystal of class 6 mm with respect to principal x_1, x_2 and x_3 (poling) axes, the constitutive relations can be written in the following form:

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{23} \\ 2\varepsilon_{31} \\ 2\varepsilon_{12} \end{Bmatrix} = \begin{bmatrix} s_{11} & s_{12} & s_{13} & 0 & 0 & 0 \\ s_{12} & s_{11} & s_{13} & 0 & 0 & 0 \\ s_{13} & s_{13} & s_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & s_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & s_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & s_{66} \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{12} \end{Bmatrix} + \begin{bmatrix} 0 & 0 & d_{31} \\ 0 & 0 & d_{31} \\ 0 & 0 & d_{33} \\ 0 & d_{15} & 0 \\ d_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix} \tag{A.1}$$

$$\begin{Bmatrix} D_1 \\ D_2 \\ D_3 \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & d_{15} & 0 \\ 0 & 0 & 0 & d_{15} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{12} \end{Bmatrix} + \begin{bmatrix} \epsilon_{11}^{\mathrm{T}} & 0 & 0 \\ 0 & \epsilon_{11}^{\mathrm{T}} & 0 \\ 0 & 0 & \epsilon_{33}^{\mathrm{T}} \end{bmatrix} \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix} \tag{A.2}$$

where

$$\sigma_{23} = \sigma_{32}, \quad \sigma_{31} = \sigma_{13}, \quad \sigma_{12} = \sigma_{21} \tag{A.3}$$

$$\varepsilon_{23} = \varepsilon_{32}, \quad \varepsilon_{31} = \varepsilon_{13}, \quad \varepsilon_{12} = \varepsilon_{21} \tag{A.4}$$

$$\begin{aligned} &s_{11} = s_{1111} = s_{2222}, \quad s_{12} = s_{1122}, \quad s_{13} = s_{1133} = s_{2233}, \quad s_{33} = s_{3333}, \\ &s_{44} = 4s_{2323} = 4s_{3131}, \quad s_{66} = 4s_{1212} = 2\left(s_{11} - s_{12}\right) \end{aligned} \tag{A.5}$$

$$d_{15} = 2d_{131} = 2d_{223}, \quad d_{31} = d_{311} = d_{322}, \quad d_{33} = d_{333} \tag{A.6}$$