

# An Advanced Image Reconstruction Method and Quantitative Method for Single-Photon Emission Computed Tomography: Compressed Sensing

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## Background

Compressed sensing (CS) is an attractive technology that enables the restoration of a high signal-to-noise ratio (S/N) image from scarce collected data that would normally be expected to produce an image with a low S/N. Images with many zero components are called sparse images; these can be reconstructed from fewer data than typical images. Even in cases where the image to be acquired has, in principle, few zero components, it may be advantageous to convert it to an image with many zero components by using a sparsifying transformation. For example, this approach can enable a reduction in the dose of radiopharmaceuticals, the amount of contrast agent, or radiation in medical procedures such as X-ray computed tomography (CT). It also enables high-speed sequencing in magnetic resonance imaging (MRI). For MRI, when undersampling is performed in the k-space, coherent aliasing is generated, as shown Figure 1, and it is difficult to restore the signal. However, if random undersampling is performed, then incoherent aliasing is generated, as shown in Figure 2, and the signals can be restored using CS [1].

The parallel imaging (PI) method, which is currently used as a technology for high-speed MRI sequencing, can process coherent aliasing by determining equidistant sampling with sensitivity encoding (SENSE) parameters in correcting data. However, PI has the characteristic that the imaging speed varies depending on the number of coils used to collect the data. In contrast, since CS utilizes incoherent aliasing that is generated by random sampling, the imaging speed can be increased without having to add a physical device such as a coil by just changing the random configuration or the data acquisition rate. CS and PI can be used simultaneously because they are independent methods, so high speed imaging at a maximum speed of approximately 8 times the standard rate is anticipated [2].

## Theory

We explain the procedure for applying CS to MRI. The k-space data, which is the Fourier transform of the measured magnetic resonance image, are collected using an MRI scanner. CS acquires randomly sampled data from this collection and subjects it to iterative calculation. A two-dimensional MRI image has frequency encoding and phase encoding: it is imaged by filling the data line in the phase direction. The technique which is used to perform the random sampling of the phase direction data determines the image quality. At present, advanced sampling methods are based on the assumption that frequency encoding data through the origin of the frequency space have to be sampled. For example, one method samples randomly using a uniform or a Gaussian probability density function, and another method samples low-frequency data around the origin of the frequency space at a fixed rate and samples the high-frequency data randomly. Although many methods have been devised, the best method has not yet been determined. The post-processing image reconstruction is performed iteratively, swapping between the k-space with randomized data and real space with the calculated inverse Fourier transform. Iterative calculation methods include the projection onto

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convex sets (POCS) method, the fast iterative shrinkage thresholding algorithm, (FISTA) and the conjugate gradient method [1,3,4]. Using these algorithms, images with poor S/N are transformed into images with high S/N. Usually, the iteration number or a convergence value are used to end the iterative processing.

The image reconstruction method using CS is described by equation 1.

$$\arg \min_m \|F_u m - y\|_2^2 + \lambda \|\Psi m\|_1 \quad (1)$$

The vector  $m$  represents pixel values of the target image,  $y$  is the measured data in k-space,  $F_u$  is the operator for random sampling after a Fourier transform and  $\Psi$  is the sparsifying transform. Lambda ( $\lambda$ ) is a weighting coefficient controlling the relative importance of the two terms. Image reconstruction defined as finding the target image  $m$  which minimizes the weighted linear sum of two terms.

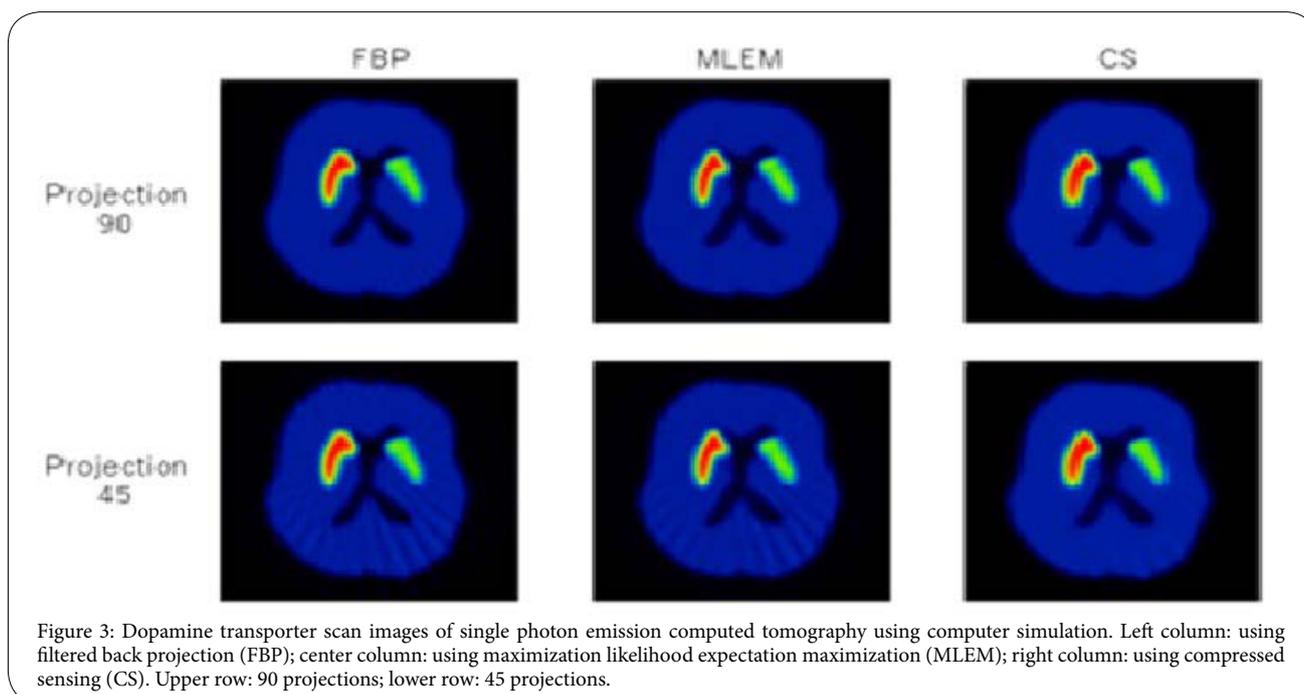
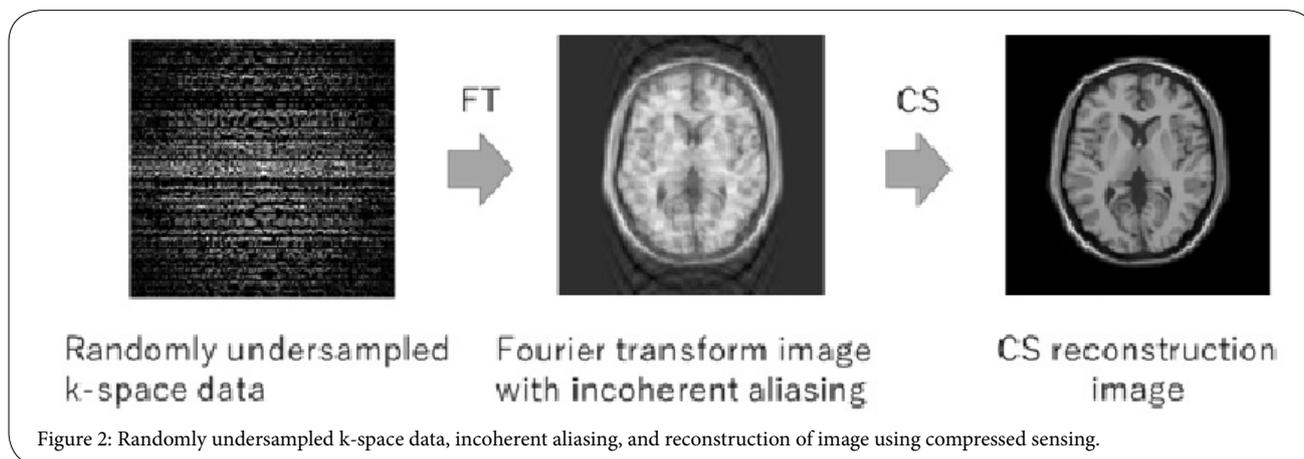
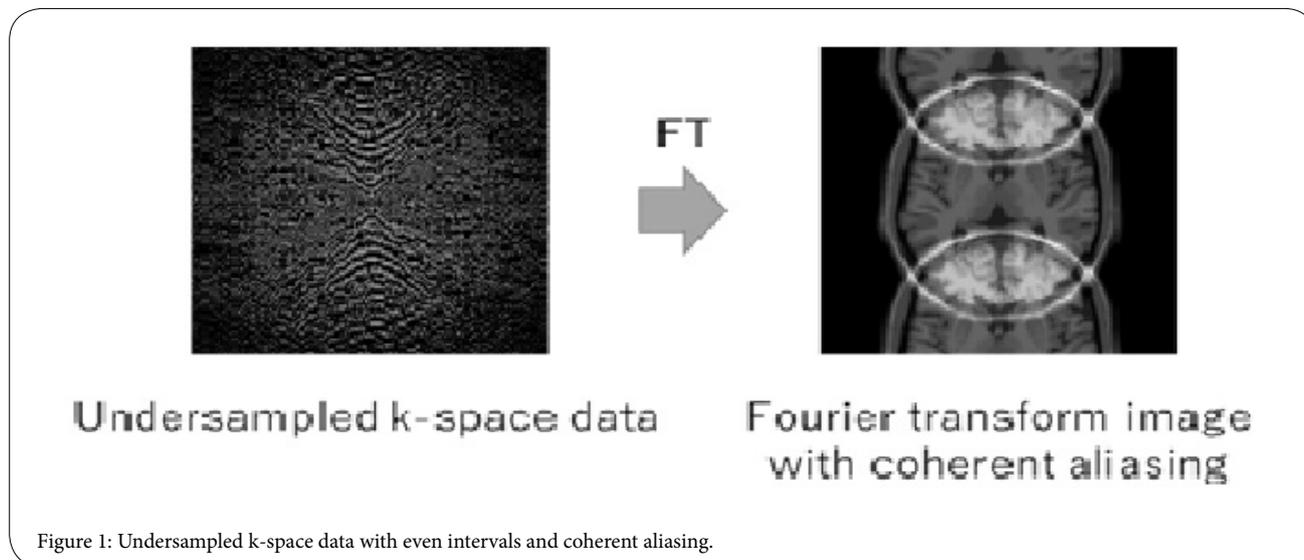
A subscript  $n$  on  $\|\cdot\|_n$  indicates an  $\ell_n$ -norm, so  $\|F_u m - y\|_2^2$  is the square of an  $\ell_2$ -norm and  $\|\Psi m\|_1$  is an  $\ell_1$ -norm. An  $\ell_2$ -norm is a measure defined as the square root of the sum of squared values.  $\|F_u m - y\|_2^2$  therefore, the sum of the squared differences between the Fourier-transformed target image with random sampling  $F_u m$  and the observed k-space image  $y$ . The minimization of this value is equal the least squares method. The  $\ell_1$ -norm in the second term of equation (1) is a measure which is the sum of absolute values. Here the values used are the components of a vector formed by applying the sparsifying transform  $\Psi$  to the target image  $m$ . The inclusion of this term is a constraint condition whose minimization indicates good results in solving the optimization problem.

As sparsity is important for images that are analyzed using CS, a sparsifying transform is recommended to increase the number of zero components. Types of sparse transforms include discrete cosine, wavelet, and curvelet transforms [5]. When we consider the wavelet

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transform, we can understand the use of the sparsifying transformation as multiresolution analysis. The image including many low-frequency components obtained by compressing the original image is converted to the second quadrant, and the other image, which includes high-frequency components, is converted to the other quadrants; therefore, there are many components close to zero. Sparsification is used not only to sparsify images, but also for denoising. Total variation (TV) is an effective method of denoising [6]. TV is a regularization that is smoothing while preserving edges. In the case of a reconstruction using TV,  $\|m\|_{TV}$  is added linearly to equation (1).

Recently, various medical device manufacturers have started to release new magnetic resonance devices employing the CS method, so the further development of CS technology is promised. In fact, a reduction of acquisition time is possible, because the amount of collected data is reduced by CS technology.

### Application to Single-photon Emission Computed Tomography (SPECT)

In nuclear medicine, if CS technology is used for corpus striatum or myocardium, it will decrease the amount of radiopharmaceuticals, reduce acquisition time, and have a variety of effects on image quality or quantity in SPECT. We performed a computer simulation of the reconstruction of a dopamine transporter (DaT) SPECT image using the CS method and show the results in Figure 3.

In Figure 3 the left column shows the results of using the filtered back projection (FBP) method, the center column shows maximization likelihood expectation maximization (MLEM) results, and the right column shows the results of using the CS reconstruction method with TV. In the upper row, the number of projections is 90, while in the lower row, the number of projections is 45. When using FBP and MLEM, streak artifacts are observed if the number of projections is reduced. However, even if the number of projections is halved, a good image with few streak artifacts is obtained by using the CS reconstruction method. In this way, using the CS reconstruction method can decrease the required number of projections while maintaining SPECT image quality. This result also indicates that the acquisition time will be shortened and the amount of radiopharmaceuticals required will be reduced.

### Competing Interests

The authors declare that no competing interests exist.

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