

Thermoelectric Cooling: The Maximum Temperature Difference and The Relation of Thermoelectric Properties with Geometric Parameters

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Abstract

For the proper performance of a thermoelectric device, are needed materials with high intrinsic merit figures, this amount is a function of the thermoelectric properties of the material. However although the bulk could present good values in their thermoelectric properties, when trying to build a thermoelectric device, other aspects must be considered. A aspect that For example for cooling, an important quantity is the maximum temperature difference that can be achieved by a thermocouple. This paper shows how this amount is related to dimensional parameters of the thermocouple. Particularly is dened the transverse area ratio of the thermocouple legs, whose value more efficient is function of the thermoelectric properties ρ and k .

Introduction

Thermoelectric eects are exploited for the operation of solid state devices known as thermoelectric modules (TEM), which are used in diverse applications [1]. For example in modern laser, optical and radioelectronic systems cannot be built without thermoelectric cooling and thermostatically controlled systems, also are needed for precision temperature control of photonics [4] components: Electronic components (memory, ASICs,) meet performance and reliability specs over wide range of operating temperatures; e.g., Intel Atom (bare die) microprocessor requires 0-85°C case (die) temperature [5].

A thermoelectric module is a device, which produce a temperature difference with current flow by means of an eect known as Peltier eect, due to this eect the TE has a similar function to that of a refrigerator. Conversely, if the device is submitted to a temperature difference, a potential difference is created which can be used to power an external load, this effect is known as Seebeck effect.

A thermoelectric device have the following advantages: no moving parts, ideal when precise temperature control is required, ability to lower temperature below ambient, heat transport controlled by current input, compact size make them useful for applications where size or weight is a constraint, ability to alternate between heating and cooling. No working fluids and gases pollutants, low noise operation [2].

The TEM 's are made of thermoelectrics material, whose quality of performance is given by a parameter known as the gure of merit. It is dened by the following equation, [6]:

$$z = \frac{\alpha^2}{\rho k} \quad (1)$$

where ρ is the electrical resistivity, k is the thermal conductivity, and α is the Seebeck Coefficient, these properties are dependent on temperature. The most basic model of a TEM is a thermocouple (a P leg matched to N leg), for which the figure of merit is given by the equation (2).

$$ZT = \frac{(\alpha_p - \alpha_n)^2}{(k_p + k_n)(R_p + R_n)} T \quad (2)$$

Publication History:

Received: May 30, 2018

Accepted: June 28, 2018

Published: June 30, 2018

Keywords:

Adiabatic temperature change, Electrocaloric effect, Ferroelectrics, Isothermal entropy change, Phase

Then a primary goal in thermoelectricity, is to get devices whose gure of merit is greater than the standard value of 1. For example , even for state of art Bi_2Te_3 which has a zT of 1.1, the eective device ZT is only about 0.7 based on the overall performance of the device as a cooler or power generator [9].

A technique for improving the design of TEM 's is to work with the equations of the theory of thermoelectricity and heat balances applied to heat exchangers, taking into account geometric or dimensional parameters of the thermoelectric module. For example transfer areas of heat sinks, thermoelement length, the number of thermocouples, the geometric ratio of the cross-sectional area.

The basic thermoelectric device is a thermocouple, which consists of a p-type and n-type semiconductor elements, or legs. Copper commutation tabs are used to interconnect legs. Thermoelectric module are made of multiple thermocouples connected electrically in series and sandwiched between two ceramic plates. The number of thermocouples may vary greatly from several elements to hundred of units. This allows to construct a TEC of a desirable cooling capacity ranging from fractions of Watts to hundreds of Watts.

For the improvement of thermoelectric devices, the following aspects should be addressed

1. Areas of each type legs need to be optimized
2. Two types of legs should have comparable properties
3. Current input to the device needs to be optimized
4. Have the capability of operating in new and broader temperature regimes,

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Citation: Almeida AV, Robles MAO, Brownz D (2018) Thermoelectric Cooling: The Maximum Temperature Difference and The Relation of Thermoelectric Properties with Geometric Parameters. Int J Metall Mater Eng 4: 142. doi: <https://doi.org/10.15344/2455-2372/2018/142>

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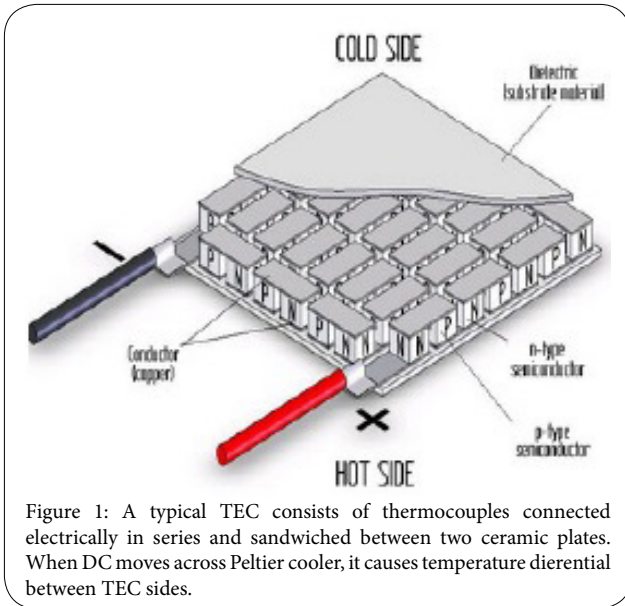


Figure 1: A typical TEC consists of thermocouples connected electrically in series and sandwiched between two ceramic plates. When DC moves across Peltier cooler, it causes temperature differential between TEC sides.

Equation Figure of Merit

This section presents the basic theory of the figure of merit for a thermocouple. The quantity of greatest importance for a refrigerator is the coefficient of performance (COP), which is defined as the ratio of heat extracted from the source to the expenditure of electrical energy [3].

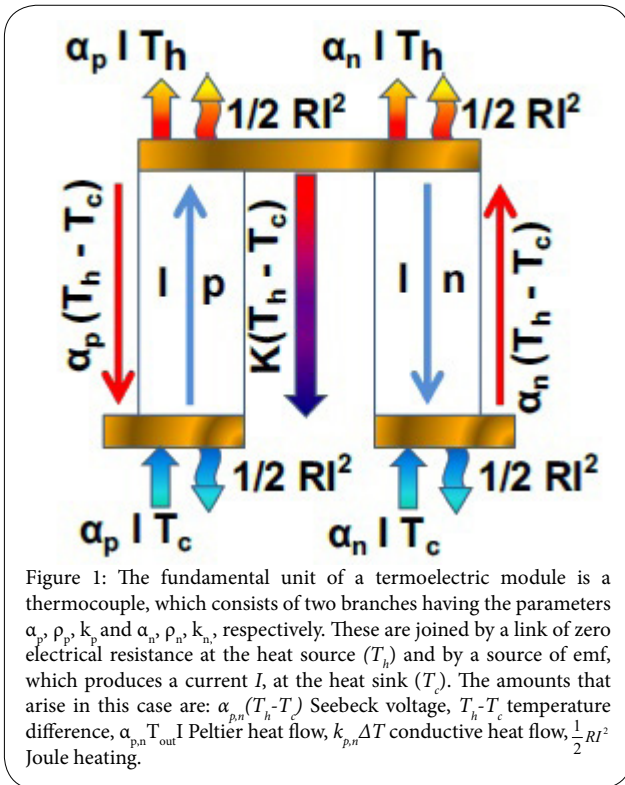


Figure 1: The fundamental unit of a thermoelectric module is a thermocouple, which consists of two branches having the parameters α_p, ρ_p, k_p and α_n, ρ_n, k_n , respectively. These are joined by a link of zero electrical resistance at the heat source (T_h) and by a source of emf, which produces a current I , at the heat sink (T_c). The amounts that arise in this case are: $\alpha_p (T_h - T_c)$ Seebeck voltage, $T_h - T_c$ temperature difference, $\alpha_p I T_c$ Peltier heat flow, $k_p \Delta T$ conductive heat flow, $\frac{1}{2} RI^2$ Joule heating.

The expression for the cooling power is

$$q_1 = (\alpha_p - \alpha_n)IT_1 - (T_2 - T_1)(k_p + k_n) - \frac{I^2(R_p + R_n)}{2} \quad (3)$$

where

$$R = \frac{L_p \rho_p}{A_p} + \frac{L_n \rho_n}{A_n} \quad (4)$$

$$k = \frac{A_p k_p}{L_p} + \frac{A_n k_n}{L_n} \quad (5)$$

The rate of expenditure of electrical energy is:

$$w = (\alpha_p - \alpha_n)I(T_2 - T_1) + I^2(R_p + R_n) \quad (6)$$

The COP ϕ , is then given by

$$\phi = \frac{(\alpha_p - \alpha_n)IT_1 - (T_2 - T_1)(k_p + k_n) - I^2(R_p + R_n) / 2}{(\alpha_p - \alpha_n)I(T_2 - T_1) + I^2(R_p + R_n)} \quad (7)$$

The current I_ϕ , that yields the maximum coefficient of performance is given by

$$I_\phi = \frac{[T_H - T_C](\alpha_2 - \alpha_1)}{R[\sqrt{1 + ZT_M} - 1]} \quad (8)$$

At this current, the COP is given by:

$$\phi_{\max} = \frac{T_C}{T_H - T_C} \frac{\sqrt{1 + ZT_M} - T_H / T_C}{\sqrt{1 + ZT_M} + 1} \quad (9)$$

where,

$$z = \frac{(\alpha_p - \alpha_n)^2}{(k_p + k_n)(R_p + R_n)} \quad (10)$$

this quantity is known as the gure of merit of the thermocouple and may be optimised for a given pair of thermoelectric materials. The aim should be to make the product $(k_p + k_n)(R_p + R_n)$ as small as possible. This result is obtained when the form factors for the two legs satisfy the relation:

$$\frac{L_n A_p}{L_p A_n} = \left(\frac{\rho_p k_n}{\rho_n k_p} \right)^{1/2} \quad (11)$$

when (11) is satisfied, ZT is given by:

$$ZT = \frac{(\alpha_p - \alpha_n)^2}{[(k_p \rho_p)^{1/2} + (k_n \rho_n)^{1/2}]^2} T \quad (12)$$

This gure of merit is usually applied to a pair of materials, is say a thermocouple.

Maximum Temperature Difference: Comparison Between A Unileg and Thermocouple

One of the most important characteristics of a thermocouple for cooling applications is the maximum temperature difference that can be reached through the Peltier effect. This quantity, ΔT_{\max} , can be calculated, when the COP is equal zero. So that:

$$\Delta T_{\max} = \frac{1}{2} ZT_1^2 \quad (13)$$

In this section we make a comparison between the maximum temperature difference of a unileg (which is a useful resource in the design of thermoelectric systems) and the maximum temperature difference of a thermocouple (two legs of the same material), using the relationship of the thermoelectric properties ρ and k and the geometric parameters, in particular the area ratio. This step is important in the experimental practice.

The equation of the maximum temperature difference for a thermocouple, with the two legs of the same length but different area is:

$$\Delta T_{N+P}^{\max} = \frac{(\alpha_N - \alpha_P)^2 (T_{\text{cold}})^2}{2(k_{\rho\rho P} + \gamma k_{N\rho P} + \frac{k_{\rho\rho N}}{\gamma} + k_N \rho N)} \quad (14)$$

where:

$$\gamma = \frac{A_N}{A_P} \quad (15)$$

is the area ratio.

The figure (3.B and 3.C) show two cases to the equation (14), the maximum temperature difference of one thermoelectric leg plus metal wire (unileg) and the maximum temperature difference of two thermoelectric legs of same material (thermocouple), respectively, both as function of area ratio. We have used two different thermoelectric materials (a conventional and the other is a novel material) to show the usefulness of the equation (14).

Now when the thermocouple is made of two legs of the same material, both of the same area and same length (conventional model), the maximum temperature difference is given by the equation (16).

$$\Delta T_{STM}^{\max} = \frac{\alpha_{TM}^2 (T_{\text{cold}})^2}{2_{\rho TM} k_{TM}} \quad (16)$$

For the design of a thermocouple (which is the real element that composes the TEM) for cooling, the $\Delta T_{\text{unileg}}^{\max}$ is used to approach $\Delta T_{\text{Thermocouple}}^{\max}$. Then a way to make this approach is by means of the definition of the maximum temperature difference ratio, by using the equations (14,16),

$$\frac{\Delta T_{STM}^{\max}}{\Delta T_{N+P}^{\max}} = \frac{\alpha_{TM}^2 (k_{M\rho M} + k_{TM\rho M\gamma} + k_{M\rho M\gamma} - 1 + k_{TM\rho TM})}{(\alpha_M - \alpha_{TM})^2 k_{TM\rho TM}} \quad (17)$$

the equation (17) shows the relationship between the thermoelectric properties (α , ρ , k) and geometric parameters, in this case the transversal section areas of legs. The figure (4) shows the maximum temperature difference ratio as function of area ratio,

A useful technique for the purposes of measuring thermoelectric properties is to maintain the set temperature in any of the two points (cold or hot), furthermore the study of this aspect could be useful for developing an application where one of the temperatures (T_{cold} or T_{hot})

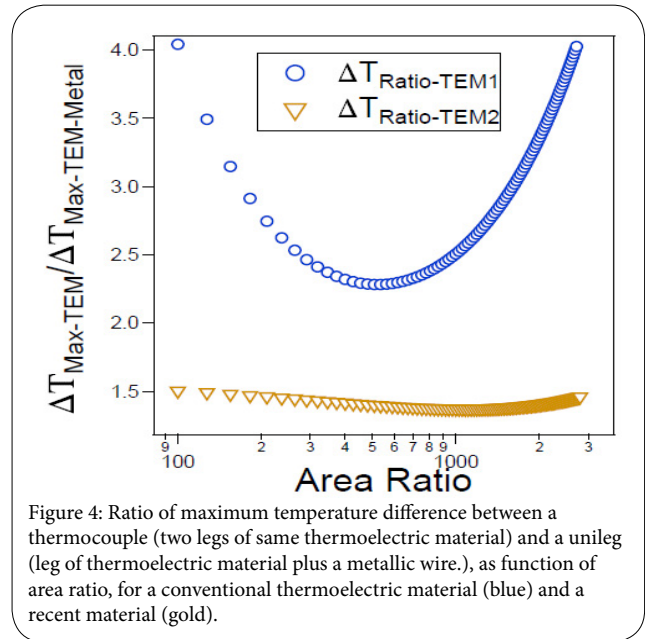


Figure 4: Ratio of maximum temperature difference between a thermocouple (two legs of same thermoelectric material) and a unileg (leg of thermoelectric material plus a metallic wire.), as function of area ratio, for a conventional thermoelectric material (blue) and a recent material (gold).

remain fixed, the figure (5.A) and figure (5.B) show the maximum temperature difference ratio as function of ZT at T_{cold} fixed and at T_{hot} fixed respectively.

To have an approximate value of the maximum temperature difference that could be achieved with a thermocouple with reference to the maximum temperature difference achieved by a unileg, we have maximized the equation (17) with respect to γ ,

$$\text{best-}\gamma = \frac{\sqrt{k_M} \sqrt{\rho TM}}{\sqrt{k_{TM}} \sqrt{\rho M}} \quad (18)$$

$$\Delta T_{\text{ratio-best-}\gamma}^{\max} = \frac{\alpha_{TM}^2 (\sqrt{k_M} \sqrt{\rho M} + \sqrt{k_{TM}} \sqrt{\rho TM})^2}{(\alpha_M - \alpha_{TM})^2 k_{TM\rho TM}} \quad (19)$$

It is noteworthy that the equation (18) shows very clearly the relationship between the thermoelectric properties and geometrical parameters of the legs, in this case (ρ , k) with (A_N , A_P).

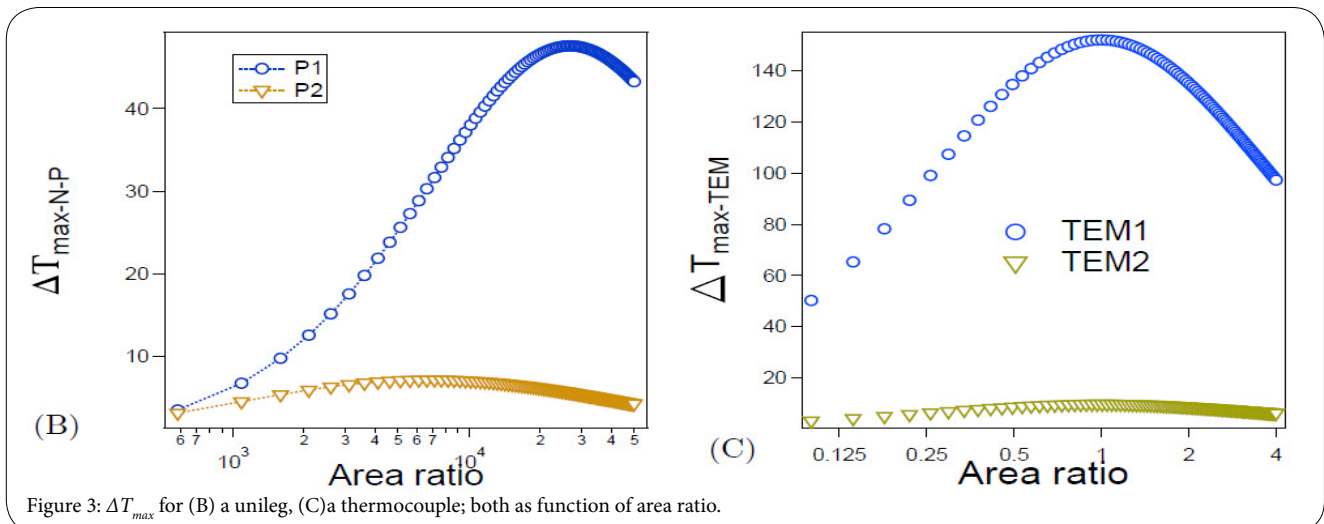
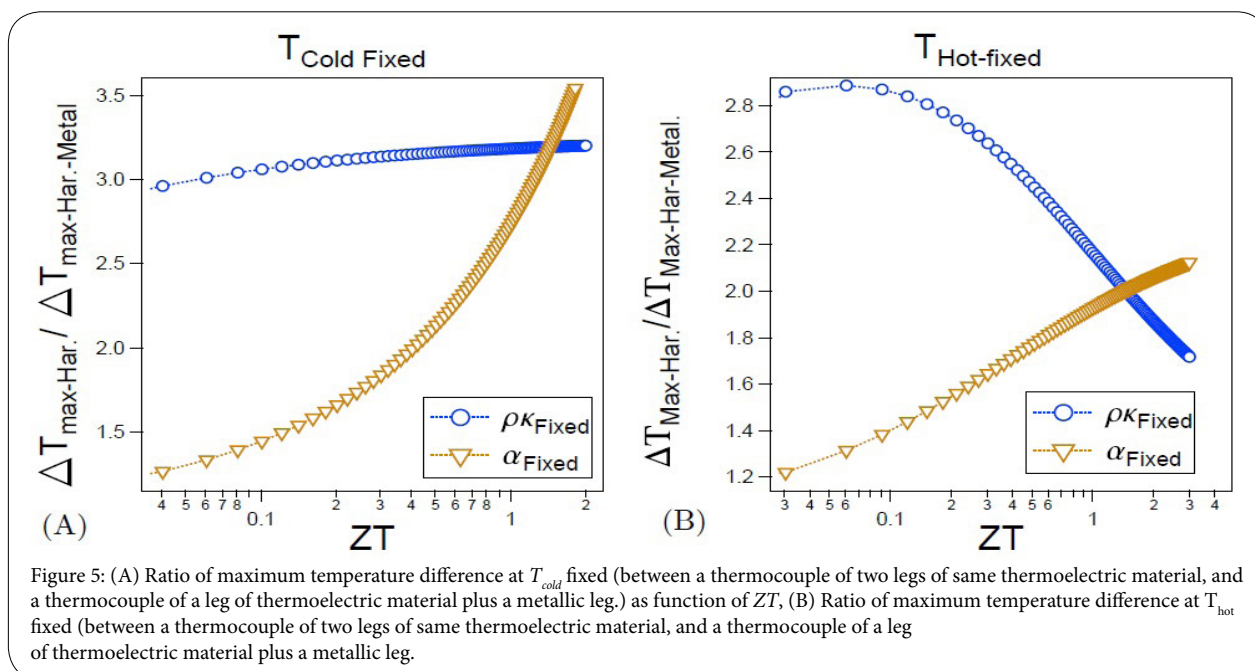


Figure 3: ΔT_{max} for (B) a unileg, (C) a thermocouple; both as function of area ratio.



Material	Seebeck coefficient ($\mu\text{V} / \text{K}$)	Resistivity ($\Omega \cdot \text{m}$)	Thermal conductivity ($\text{W} / \text{m} \cdot \text{K}$)
TM	$\alpha_n = -219 * 10^{-6}$	$\rho_n = 1.71 * 10^{-5}$	$\kappa_n = 1.36$
Cu	$\alpha_{Cu} = 1.70 * 10^{-6}$	$\rho_{Cu} = 1.71 * 10^{-8}$	$\kappa_{Cu} = 372$

Figure 6: Thermoelectric properties.

To show the usefulness of the equation (19), it is evaluated by using the numerical values for the properties of a thermoelectric material (TM) figure (6), while the metal in this case is copper (Cu).

The result is:

$$\Delta T_{ratio-best-\gamma}^{\max} = 2.28393 \quad (20)$$

this value indicates that the maximum value of the temperature difference reached by a thermocouple ΔT_{twoleg}^{\max} is approximately two times ΔT_{oneleg}^{\max} , and it provides a reference value for the design of a thermocouple starting from a unileg, when the two legs of the TEG are made of the same thermoelectric material.

Conclusion

In this work we have studied a basic aspect for the design of a conventional thermocouple for thermoelectric cooling, with reference to the model of a unileg. We have show how to approach the maximum temperature difference of a thermocouple (two legs of the same material) by mean of the maximum temperature difference of a unileg, by definition of the maximum temperature ratio $\frac{\Delta T_{STM}^{\max}}{\Delta T_{N+P}^{\max}}$,

this quantity have been maximized regarding to the area ratio γ , giving as result 2:28393, is say $\Delta T_{Thermocouple}^{\max} \approx 2\Delta T_{Unileg}^{\max}$. Also the equation

(17) maybe useful when T_{cold} or T_{hot} are red. Finally in this work have been show a relationship of the thermoelectric properties (ρ , k) with the geometric parameters (A_N , A_P), by mean of the area ratio γ .

Acknowledgments

We thank to CONACYT for supporting the academic stay at CALTECH.

Competing Interests

The authors have no competing interests with the work presented in this manuscript.

References

- Bell LE (2008) Cooling, Heating, Generating Power, and Recovering Waste Heat with Thermo-electric Systems. Science 321: 1457-1461.
- DiSalvo FJ (1999) Thermoelectric Cooling and Power Generation. Science AAAS (New York, N.Y.) 285: 703-706.
- Goldsmid HJ (2009) Theory of Thermoelectric Refrigeration. Springer Series in Materials Science.
- Gromov G. Thermoelectric Cooling Modules. BUSINESS BRIEFING: GLOBAL PHOTONICS APPLICATIONS AND TECHNOLOGY.
- Hodes Ma (2011) Thermoelectric Modules : Principles and Inter PACK 2011 Tutorial. InterPACK 2011 Tutorial.
- Snyder GJ (2008) Small Thermoelectric Generators. The Electrochemical Society Interface.

7. Shakouri A (2011) Recent Developments in Semiconductor Thermoelectric Physics and Materials. Annual Review of Materials Research
8. Harman TC, Taylor PJ, Walsh MP, LaForge BE (2002) Quantum dot superlattice thermoelectric materials and devices. Science (New York, N.Y).
9. Snyder GJ, Toberer ES (2008) Complex thermoelectric materials. Nature materials.
10. Tajima K, Tanaka K (2003) Thermoelectric Material, Thermoelectric Element, Thermoelectric Module and Method for Manufacturing the Same. Kyocera Corporation.
11. Heikes RR, W. Ures R Jr (1961) Thermoelectricity: Science and Engineering. Inter-science Publishers.
12. Mahan G, Sales B, Sharp J (1997) Thermoelectric Materials: New Approaches to an Old Problem. Phys. Today.

This article was originally published in a special issue:

Thermoelectric Materials Properties and Performances

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