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The Q-generalized Probability Weighting Function and the Q-exponential Probability Discounting Function

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Abstract

Decision under risk has been studied in economics and psychology in a relatively independent manner. This study demonstrates a mathematical connection between the probability weighting function in behavioral economics and the probability discounting function in behavioral psychology, by utilizing the q-generalized exponential and logarithmic functions developed in Tsallis non-extensive thermostatistics. Implications for psychophysical neuroeconomics are discussed.

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Introduction

Decision under risk and uncertainty has first been systematically modelled mathematician von Neumann and economist Morgenstern [9] in an expected utility theory. However, later studies in behavioral economics experimentally revealed that human decision making under risk and uncertainty does not follow von Neumann and Morgenstern's expected utility theory [1,2]. To model this violation of the expected utility theory, Kahneman and Tversky[2] introduced nonlinear probability weighting functions for modelling overweighing of small probabilities and underweighing of large probabilities. Moreover, Prelec[3] defined the precise probability weighting function to model experimental data of human decision making under risk.

In behavioral psychology as well, decision under risk has been studied. The main approach is to establish the mathematical formula describing human and animal choice under risk by exploiting the mathematical equivalence between an increase in delay until receipt and a decrease in probability in repeated gambles [4]. However, with few exceptions [6,7], the link between economic and psychological approaches have not appropriately been explored. In this study, we demonstrate the link by utilizing the q-generalizations of logarithmic and exponential functions.

The q-generalized probability weighting function and the q-exponential probability weighting function

In behavioral economics of decision under risk [2], the probability weighting function [3] has been introduced to model human decision making under risk which violates von Neumann and Morgenstern's expected utility theory [9]. The well-established Prelec's probability weighting function is defined as

$$w(p) = 1/\exp(\beta(-\ln(p))^{\alpha})$$

By utilizing this probability weighting function, human decision making under risk can be modeled better than the expected utility theory [3].

Here we introduce a q-logarithmic function (a type of generalized logarithmic functions):

$$ln_q(x) = (x^{(1-q)} - 1)/(1-q)$$

where

$$q \to 1 \Rightarrow \ln_q(x) = \lim_{q\to 1}\frac{1}{1-q}\left(x^{1-q}-1\right) = \ln(x),$$

$$q = 2 \Rightarrow \ln_q(x) = -\left(\frac{1}{x}-1\right).$$

We now introduce the following q-generalized probability weighting function:

$$w_q(p) = 1/\exp\left(\beta\left(-\ln_q(p)\right)^{\alpha}\right)$$

$$= 1/exp\left(\beta\left(-\left(\frac{1}{1-q}\right)\left(p^{1-q}-1\right)\right)^{\alpha}\right).$$

It is to be noted that when $q\to 1$, the Prelec's probability weighting function is recovered:

$$w_1(p) = \lim_{q\to 1} 1/\exp\left(\beta\left(-\ln_q(p)\right)^{\alpha}\right)$$

$$= 1/\exp\left(\beta\left(-\ln(p)\right)^{\alpha}\right) = w(p).$$

Furthermore, if we fix q=2 and α =1, we obtain

$$w_2(p) = 1/\exp\left(\beta\left(-\left(\frac{1}{1-2}\left(p^{1-2}-1\right)\right)\right)^{1}\right)$$

$$= 1/\exp\left(\beta\left(-\left(-\frac{1}{p}+1\right)\right)\right)$$

which is the same as the exponential probability discounting function [4]. Please note that $\frac{1}{p}-1$ is the "odds against"corresponding to waiting time in repeated gambles. If this probability discounting function is further generalized with the q-exponential function $(\exp_q(x) := (1+(1-q)x)^{\frac{1}{1-q}})$, we obtain the doubly q-generalized probability weighting function

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$$w_{2r}(p) = 1/\exp_r\left(\beta\left(\frac{1}{p}-1\right)\right)$$

which is the same as Takahashi's q-exponential probability weighting function [5] fitting well to experimental data [7,8] (we here utilize r instead of q for the generalization parameter to avoid confusion with the q-parameter appearing in equation 3). Also, when r is fixed at 0, we obtain Rachlin's hyperbolic probability weighting function [4]:

$$w_{20}(p) = 1/\left(1+\beta\left(\frac{1}{p}-1\right)\right).$$

Discussion

As is demonstrated, the probability weighting function in behavioral economics and the probability discounting function in behavioral psychology are closely linked through q-generalized functions developed in Tsallis non-extensive thermostatistics [10]. Since parameters in the q-exponential discounting function are related to neurobiological and psychophysical information processing associated with decision making under risk [7], future studies in applied mathematics and neuroeconomics should explore mathematical principles underlying these neural computations.

Competing Interests

The authors declare no competing interests.

Author Contributions

Both the authors substantially contributed to the study conception.

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