Injan et al., Int J Appl Exp Math 2017, 2: 114
http://dx.doi.org/10.15344/ijaem/2017/114

International Journal of
Applied & Experimental Mathematics

Research Article Open Access

A Study of Tunable Coefficient in Whitecapping Formulation for Wave Height Over the Gulf of Thailand

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Abstract

The aim of this research was to study of tunable coefficient in whitecapping formulation of Komen and Jansen for wave height over the gulf of Thailand in the Simulating Wave Nearshore (SWAN) model on a tropical storm Podul during a period of 13~17 November 2013. The wave height from adjusting the tunable coefficient in the whitecapping formulation were compared with the Jason-2 satellite data. In addition, the result of whitecapping from Komen and Janssen were different. It was found that the tunable coefficient in whitecapping formulation of Komen was optimized for simulation of the wave height over the gulf of Thailand with the tunable coefficient in whitecapping formulation of Komen and Janssen were C_{ds} = 3.0x10^{-5} and 6.6 respectively with the skill score of 0.7138.

Publication History:

Received: December 02, 2016
Accepted: March 27, 2017
Published: March 29, 2017

Keywords:

Gulf of Thailand, Tropical Storm Podul, SWAN model, Wave height

Introduction

The gulf of Thailand is a tourist attraction and it is an important economic maritime source of Thailand. The long-term data of wind are necessary for the various applications for the sustainable development of marine economy, including coastal structure design, transportation and sediment erosion of coastal and marine pollution [1,2]. The characteristic of wave can be estimated using a numerical method such as near shore wave rather than measured data, due to the lack of measurement of the waves in a wide area [3]. To forecast, wave height is very important for safe living of people in the coastal areas connected with human activities in the sea.

The SWAN (Simulating Wave Nearshore) model is widely used for the military planning of the Navy, building design around the coastal areas, studying the coastal change and protecting against maritime disasters [4]. The model was a recognized model applying in the wave simulation that used the wind data at 10 m about the sea level for the initial data in the wave simulation. The aims of this research are the operational forecasting of the waves by the model over the gulf of Thailand and inspection of the model to precisely simulate waves near the coast.

Track of tropical storm Podul

The tropical storm Podul formed on 9 November 2013 and dissipated on 16 November 2013. It was a thunderstorm away from Palau around 1,175 kilometers on 9 November 2013 and developed into a storm which moved from the northeast to the southwest. On 10 November 2013, it intensified into a tropical depression as it moved to Palau and the Mindanao Island, Philippines on 11 November 2013, it weakened and moved to the south of Palau. It passed the Philippines into the Sulu sea (on 12 November 2013), Palawan islands and the South China Sea (on 13 November 2013) and it intensified into a tropical storm with a maximum wind speed of 65 km/h away from the coastal area of Vietnam around 275 kilometers. The tropical storm moved towards the southwest from the Vietnam coast and entered into the gulf of Thailand on 15 November 2013, and the Malay Peninsula on 16 November 2013. Finally, it moved to the southeast of India and dissipated. The track of tropical storm Podul was shown as Figure 1.

Figure 1: The track of tropical storm Podul.
(http://www.naewna.com/local/77638)

The description of models

SWAN model

The SWAN model is a third-generation numerical wave model by Delft University of Technology in the Netherlands and designed specifically for wave in coastal areas. It is based on the wave action balance equation [5]. The SWAN model is capable of predicting the wave height in the gulf of Thailand [6,7]. The spectral action balance equation for coordinates is given by

$$\frac{\partial}{\partial t}N+\frac{\partial}{\partial x}(c_x N)+\frac{\partial}{\partial y}(c_y N)+\frac{\partial}{\partial \sigma}(c_\sigma N)+\frac{\partial}{\partial \theta}(c_\theta N)=\frac{S(\sigma,\theta)}{\sigma}$$

where N is the action density, x and y are longitude and latitude respectively, t is time, σ is the absolute frequency in the absence of currents, θ is wave direction of a wave component, c_x and c_y are absolute propagation velocity of wave energy in x and y direction, respectively, c_σ and c_θ are propagation velocity of wave energy in σ-space and θ-space, respectively. The term, $S(\sigma,\theta)$ on the right-hand side on the right-hand side of Eq. (1) is the source term in terms of

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energy density. In the shallow water,

$$S(\sigma,\theta) = S_{in}(\sigma,\theta) + S_{ds,b}(\sigma,\theta) + S_{ds,br}(\sigma,\theta) + S_{ds,w}(\sigma,\theta) + S_{nl3}(\sigma,\theta) + S_{nl4}(\sigma,\theta), \quad (2)$$

where S_{in} represents the generation of wave energy by wind, $S_{ds,b}$ is the bottom friction, $S_{ds,br}$ is the depth-induced wave breaking, $S_{ds,w}$ is the whitecapping, S_{nl} is a nonlinear transfer of wave energy transfer due to conservative nonlinear wave-wave interaction (both quadruplet interactions S_{nl4} and triad interactions S_{nl3}).

Whitecapping

The process of the wave energy which is lost through deep-water wave breaking is the whitecapping dissipation source term. It is primarily controlled by the steepness of the waves and it is perhaps the least understood mechanism in deep water. The whitecapping formulations that are used based on the pulse-based model of [8] and adapted from the WAMDI group [9] in the operating ocean and shelf sea wave model [10,11]:

$$S_{ds,w}(\sigma,\theta) = -\Gamma \tilde{\sigma} \frac{k}{\tilde{k}} E(\sigma,\theta)$$

where Γ is the steepness dependent coefficient, $\tilde{\sigma}$ is the mean frequency, k is the wave number and $\tilde{k}$ is the mean wave number. The steepness dependent coefficient has been slightly adapted by Janssen [12] it can be expressed as

$$\Gamma = C_{ds}\left((1-\delta) + \delta \frac{k}{\tilde{k}}\right)\left(\frac{\tilde{s}}{\tilde{s}_{PM}}\right)^m$$

where C_{ds} and δ are tunable coefficients in whitecapping formulation of Komen and Janssen respectively, is the wave number, $\tilde{k}$ is the mean wave number, $\tilde{s}$ is the overall steepness of the wave, $\tilde{s}_{PM}$ is the value of $\tilde{s}$ for the Pierson-Moskowitz spectrum [13] and m is a tunable exponent. The values of the tunable coefficients C_{ds},δ and m in this model have been obtained by closing the energy balance of the idealized wave growth conditions for deep water.

Tunable coefficients in whitecapping formulation of Komen

Komen [14] considers the energy transfer equation for well-developed ocean waves under the influence of wind, and study the conditions for the existence of an equilibrium solution in which wind input, wave-wave interaction and dissipation balance. The wind input is taken from the parameterization proposed which was based on the measurements in the Bight of Abaco and which agrees with Miles's theory. Hasselmann [8] gives the wave-wave interaction which is computed with an algorithm.The dissipation is less well-known, but it is made on the general assumption that it is quasi-linear in the wave spectrum with a factor coefficient depending only on frequency and integral spectral parameters.The assumption is the equilibrium spectrum exists was given by the Pierson-Moskowitz spectrum [13] with a standard type of angular distribution leads to a reasonable dissipation function. Due to whitecapping, assessment methods of the distribution of wave energy assume the length scale between whitecaps and waves is large and weak in the mean, resulting in a linear dissipation source term. Komen [14] obtained by closing the energy balance of the ideal state of a growing wave [15] and it is defined defined $C_{ds} = 2.36\text{x}10^{-5}$, δ = 0, and m = 4. This formula tends to underestimate the frequency and maximum wave height and wave steepness, while the more energy assessment is over to retrieve the short term. Komen can be predicted with accuracy near Hawaii and the Antarctic Ocean. Janssen developed the alternative formula for the whitecapping [5, 16].

Tunable coefficients in whitecapping formulation of Janssen

Janssen [17,18]study the resonant wave-mean flow interaction using the quasi-linear theory of wind-wave generation. It is an extension of the Miles shear flow instability in that the effect of the gravity waves on the mean wind profile is taken into account as well. The direct effect of air turbulence on the mean wind profile is modeled by a mixing length model. The results of the numerical calculation of the steady state wind profile for giving wave spectra are found to be sensitive for the parameterization of the high-frequency tail of the wave spectrum. These values are defined by closing the energy balance and they are assumed the length between wave and high frequency waves. The wind input of Janssen formula is the form of the wave energy distribution may depend more on the wave number than a linear relationship Janssen [18]. Ris [15] found that the distribution of high frequency and low frequency spread spectrum, resulting in more realistic values than the Komen formula for the wind input and whitecapping. The coefficients to tune are generally a function of C_{ds} and of the wave steepness. Applying Janssen wind formulation the tunable coefficient was introduced in term of C_{ds} and $\tilde{s}_{PM}$ [19] which is given by

$$C_{ds1} = C_{ds}\left(\frac{1}{\tilde{s}_{PM}}\right)^4$$

where C_{ds1} is coefficient for determining the rate of whitecapping dissipation. Janssen [17,18] adjusted the coefficient for the whitecapping of Janssen formula included $C_{ds}= 4.10\times10^{-5}$, δ=0.5 and m = 4. This method of dissipation depends on the steepness of the wave spectrum at and below a particular frequency.

Methodology

Wave simulation in the tropical storm Podul

This research is studied the wave height over the gulf of Thailand using the SWAN model during 15 November 2013 at 0000 UTC to 18 November 2013 at 0000 UTC as shown in Figure 2. The area covers on 6-14°N in latitude and 99-103 °E in longitude. Actually, the tropical storm Podul moves through the gulf of Thailand during 15 to 16 November 2013.

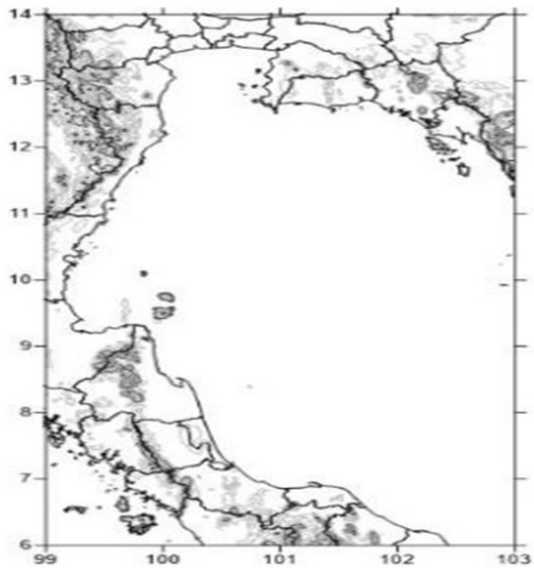

Figure 2: The area of wave simulation of the SWAN model over the gulf of Thailand.

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Information of modeling

Simulating Wave Nearshore

This study used the SWAN model to simulate the waves near the coast version 41.01 using the depth of the sea surface (topography) and the wind speed at 10 meters above the sea surface wind.

Earth topography

The earth topography (ETOPO)data show the topography and the depth of the sea floor. It has been released by the National Geophysical Data Center (NGDC), which is part of the agency National Oceanic & Atmospheric Administration (NOAA) [20]. The purpose of this data is to support and disseminate the geophysical data.This study is using the data from the ETOPO1 with the resolution is 0.0167° × 0.0167° degrees (or 1.85 x 1.85 kilometers).

Wind speeds at 10 meters above the sea surface

The wind speed data is 6-hourly wind speed data at 10 meters above the sea surface with a resolution is 0.5° × 0.5° degrees (or 55x55 kilometers) from the model Navy Global Environmental Model (NAVGEM)[21]. It has been published by the Fleet Numerical Meteorology and Oceanography Center (FNMOC) of the US Navy.

Measurements from satellite

To compare the wave height between the results from the SWAN model with the measurements from the Jason-2 satellite [22] that moved through the gulf of Thailand during storm Podul. The first time, it moved through the gulf of Thailand on 15 November 2013 at 2242 UTC and it recorded 91 points. The second time, it moved through the gulf of Thailand on 16 November 2013 at 1100 UTC and it recorded 45 points. The third time, it moved through the gulf of Thailand on 17 November 2013 at 1754 UTC and it recorded 81 points, as shown in Figure 3 for evaluating the accuracy of the SWAN model.

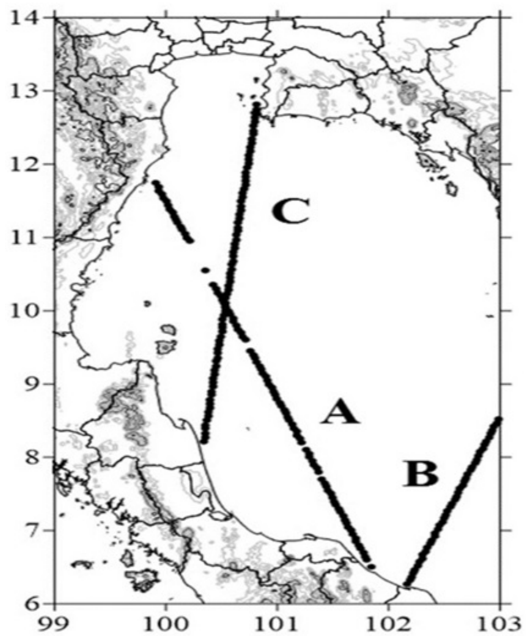

Figure 3: The movement of Jason-2 satellite over the gulf of Thailand, where line A is on 15 November 2013, line B is on 16 November 2013 and line C is on 17 November 2013.

Performance evaluation of the model

A large sample of the forecast-observation should be made with a skill score metric for the score calculations to be statistically robust. A sample of forecasts for a single predict typically includes forecasts made on a number of different dates such as the temperature at one location, or a single stock value. The sample could also pool forecast-observation across space, for a forecast made on a single date, as in the forecast of a weather event that is verified at many locations[23, 24]. The skill score is defined as:

$$S = 1 - \frac{\sqrt{\frac{1}{N}\sum_{i=1}^{N}(P_i - O_i)^2}}{\sqrt{\frac{1}{N}\sum_{i=1}^{N}O_i^2}}$$

where S is skill score, N is the number of observations, 0 is the observed values and P is the predicted values.

Results

Results from the SWAN Model with the tunable coefficient (C_{ds}) of Komen

According to compare the wave height between the model and measurement from Jason-2 satellite over the gulf of Thailand in the tropical storm Podul during 15 to 17 November 2013, it found that the tunable coefficients of Komen are 1.5×10^{-5}, 4.9×10^{-5} and 3.0×10^{-5} as shown in Figure 4, 5 and 6, respectively. The wave height calculated by the model were similar and same direction with the measurement. The correlation coefficients are 0.7146, 0.3778 and 0.5479, respectively.

Results from the SWAN Model with the tunable coefficient of Janssen

According to compare the wave height between the model and measurement from Jason-2 satellite over the gulf of Thailand in the tropical storm Podul during 15 to 17 November 2013, it found that the tunable coefficients of Janssen are 4.3, 8.6 and 6.6 as shown in Figure 7, 8 and 9, respectively. The wave height calculated by the model were similar and same direction with the measurement. The correlation coefficients are 0.7701, 0.3634 and 0.5429, respectively.

The tunable coefficient of Komen	15 Nov 2013	16 Nov 2013	17 Nov 2013
$C_{ds} = 1.5 \times 10^{-5}$	0.666983	0.590907	0.677797
$C_{ds} = 3.0 \times 10^{-5}$ (default)	0.636254	0.666954	0.716796
$C_{ds} = 4.9 \times 10^{-5}$	0.589961	0.684966	0.696234

Table 1: The statistical comparison of the wave height simulation with the tunable coefficient of Komen from the SWAN model with the measurement from satellite.

Performance and Evaluation of SWAN Model

The skill score method is analyzed for the accuracy of the SWAN model with the tunable coefficient of Komen. The default of the tunable coefficient of Komen was 3.0×10^{-5}. The skill score was 0.636. After to study the skill score technique, the highest skill score was 0.666 on 15 November 2013. It found that the tunable coefficient of Komen is 1.5×10^{-5}. On 16 November 2013, the highest skill score is 0.684 which the tunable coefficient of Komen is 4.9×10^{-5}. On 17 November 2013, the highest skill score is 0.716 which the tunable coefficient of Komen is 3.0×10^{-5} as shown in Table 1. It is more accuracy than the default.

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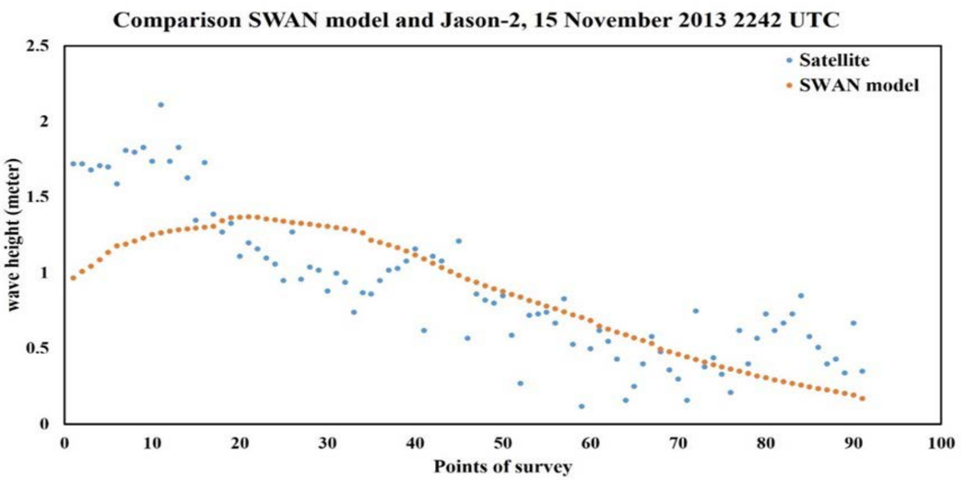

Figure 4: The wave height over the gulf of Thailand on 15 November 2013 from the SWAN model using tunable coefficient in whitecapping formulation of Komen (1984).

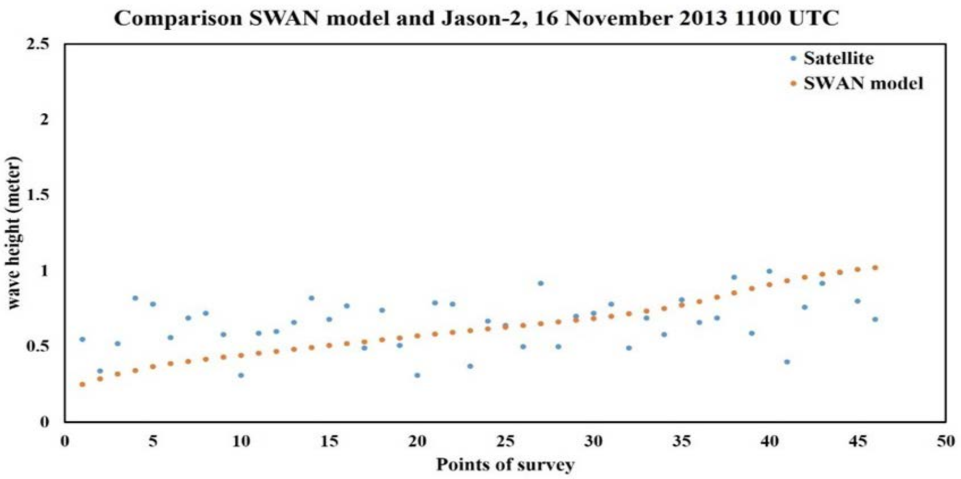

Figure 5: The wave height over the gulf of Thailand on 16 November 2013 from the SWAN model using tunable coefficient in whitecapping formulation of Komen (1984).

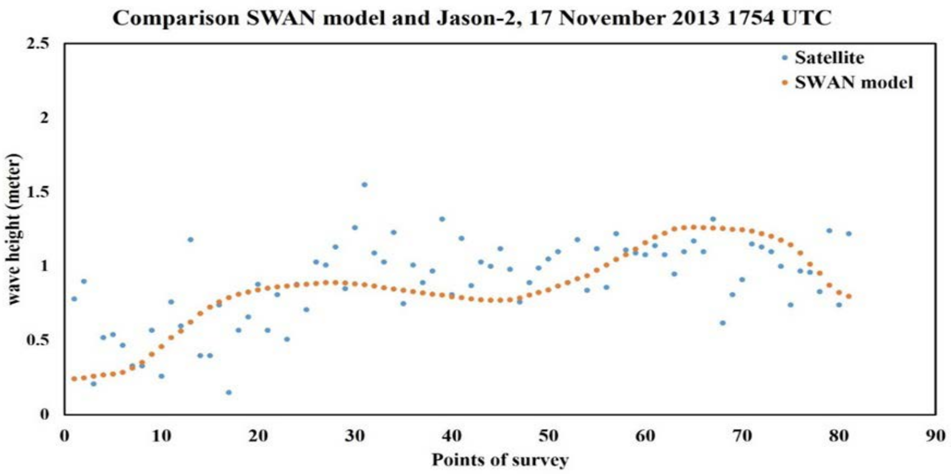

Figure 6: The wave height over the gulf of Thailand on 17 November 2013 from the SWAN model using tunable coefficient in whitecapping formulation of Komen (1984).

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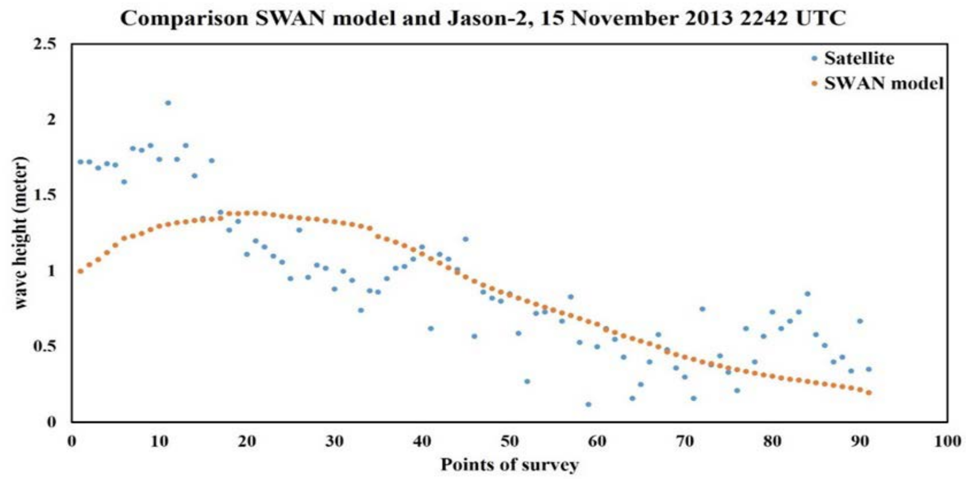

Figure 7: The wave height over the gulf of Thailand on 15 November 2013 from the SWAN model using tunable coefficient in whitecapping formulation of Janssen (1989).

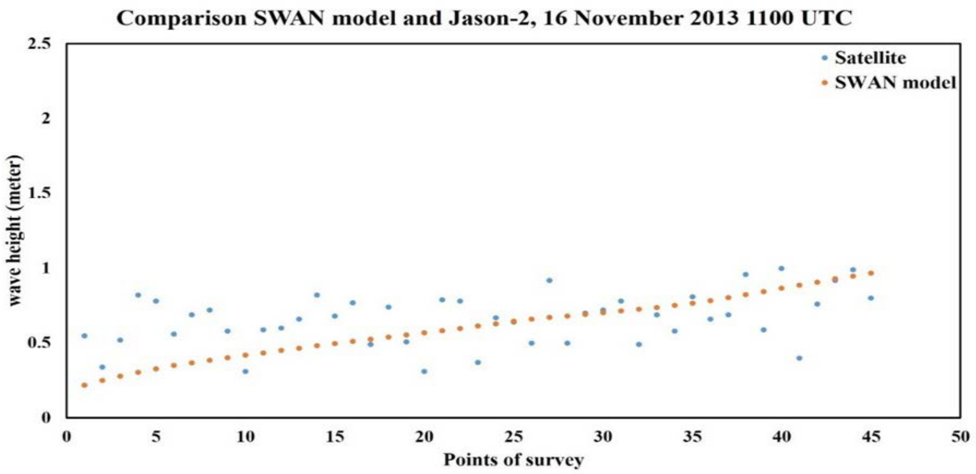

Figure 8: The wave height over the gulf of Thailand on 16 November 2013 from the SWAN model using tunable coefficient in whitecapping formulation of Janssen (1989).

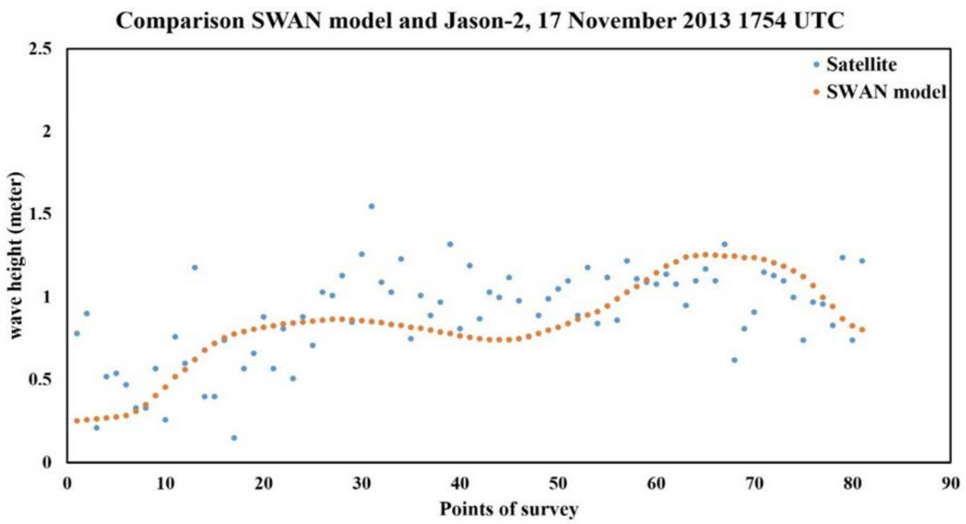

Figure 9: The wave height over the gulf of Thailand on 17 November 2013 from the SWAN model using tunable coefficient in whitecapping formulation of Janssen (1989).

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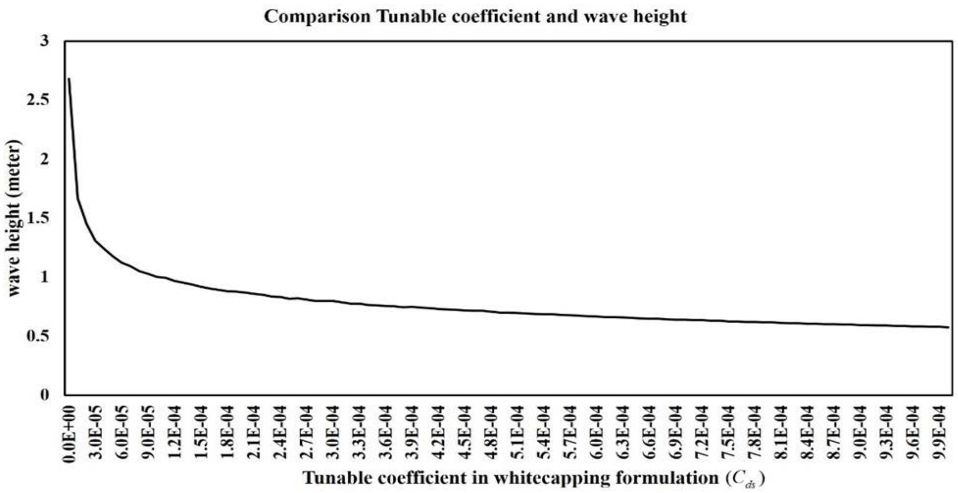

Figure 10: The tunable coefficient of whitecapping as a function of wave height to Komen (1984) on 17 November 2013, at Point 35.

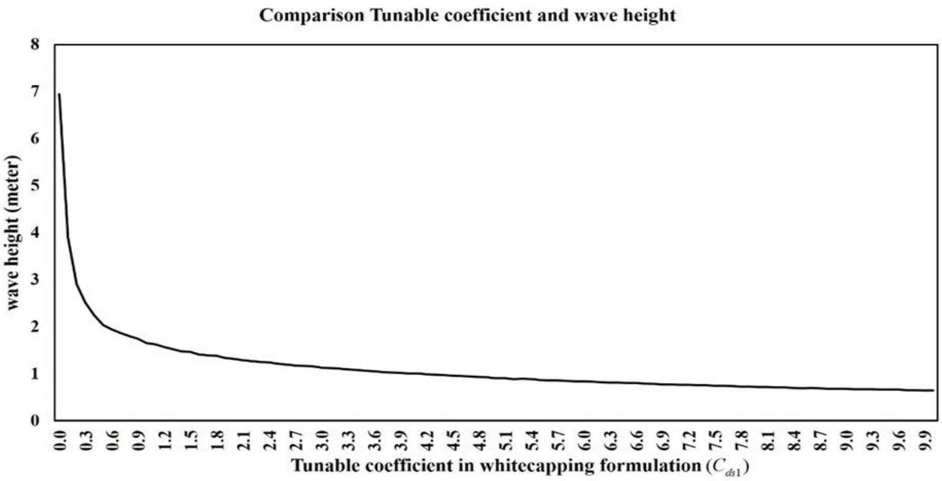

Figure 11: The tunable coefficient of whitecapping as a function of wave height to Janssen (1989, 1991) on 17 November 2013, at Point 35.

The skill score method is analyzed for the accuracy of the SWAN model with the tunable coefficient of Janssen. The default of the tunable coefficient of Janssen was 4.5. The skill score was 0.6783. After to study the skill score technique, the highest skill score was 0.6798 on 15 November 2013. It found that the tunable coefficient of Janssen is 4.3. On 16 November 2013, the highest skill score is 0.6785 with the tunable coefficient of Janssen is 8.6. On 17 November 2013, the highest skill score is 0.7138 with the tunable coefficient of Janssen is 6.6 as shown in Table 2. It is more accuracy than the default.

The tunable coefficient of Janssen	15 Nov 2013	16 Nov 2013	17 Nov 2013
C_{ds1} = 4.5 (default)	0.6783	0.5440	0.6547
C_{ds1} = 4.3	0.6798	0.5328	0.6434
C_{ds1} = 6.6	0.6397	0.6525	0.7138
C_{ds1} = 8.6	0.5862	0.6785	0.6953

Table 3: The statistical comparison of the wave height simulation with thetunable coefficient of Janssen from the SWAN model with the measurement from satellite.

Tunable coefficient in whitecapping formulation

The wave height of the tunable coefficient (C_{ds}) of Komen in the whitecapping formulation on 17 November 2013 was studied at the thirty-fifth point. It found that if the tunable coefficient is high, the wave height is low. On the other hand, if the tunable coefficient is low, the wave height is high, as shown in Figure 10.

The wave height of the tunable coefficient of Janssen in the whitecapping formulation on 17 November 2013 was studied at the thirty-fifth point. It found that if the tunable coefficient is high, the wave height is low. On the other hand, if the tunable coefficient is low, the wave height is high, as shown in Figure 11.

Conclusion

This research studied a tunable coefficient in whitecapping formulation of Komen (1984) and Janssen (1989, 1991) of SWAN model for wave simulating over the gulf of Thailand. The wave heights from Komen and Janssen were compared with the wave height from the satellite. The wave height increases gradually about 1.0 to 1.8

meters over the upper gulf of Thailand on 15 November 2013 at 0600 UTC, and moved toward the lower gulf of Thailand on 16 November 2013. It decreases gradually to around 0.5 to 1.2 meters over the lower gulf of Thailand. The wave height increase around 1.1 to 1.6 meters over the lower gulf of Thailand on 17 November 2013.The tropical storm Podul, which moved through the gulf of Thailand, is during 15 to 17 November 2013. It was found that the tunable coefficient in whitecapping formulation of Komen was optimized for simulating of the wave height over the gulf of Thailand which the skill score was 0.66 to 0.71.The tunable coefficient in whitecapping formulation of Janssen was 6.6 to 8.6 which was optimized for simulating of the wave height over the gulf of Thailand with skill score of 0.64 to 0.71.

The difference of wave height between the SWAN model and the satellite can cause from the several reasons such as from the initial wind data that obtain from NAVGEM has resolution 0.5 degree were bigger than the model resolution has 0.25 degree.

Competing Interests

The authors declare that they have no competing interests.

Acknowledgements

The authors would like to acknowledge the Navy Global Environmental Model (NAVGEM) for the data set, and the Simulating Wave Nearshore (SWAN)model.

Funding

This research was partially supported by the Faculty of Science, King Mongkut's University of Technology Thonburi.

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